# NAG Toolbox for MATLAB

# f07ja

## 1 Purpose

f07ja computes the solution to a real system of linear equations

$$AX = B,$$

where $A$ is an $n$ by $n$ symmetric positive-definite tridiagonal matrix, and $X$ and $B$ are $n$ by $r$ matrices.

## 2 Syntax

```
[d, e, b, info] = f07ja(d, e, b, 'n', n, 'nrhs_p', nrhs_p)
```

## 3 Description

f07ja factors $A$ as $A = LDL^{\mathrm{T}}$. The factored form of $A$ is then used to solve the system of equations.

## 4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

## 5 Parameters

### 5.1 Compulsory Input Parameters

1: **d**$(*)$ – **double array**

**Note**: the dimension of the array **d** must be at least $\max(1, \mathbf{n})$.

The $n$ diagonal elements of the tridiagonal matrix $A$.

2: **e**$(*)$ – **double array**

**Note**: the dimension of the array **e** must be at least $\max(1, \mathbf{n} - 1)$.

The $(n - 1)$ subdiagonal elements of the tridiagonal matrix $A$.

3: **b**(**ldb**,$*$) – **double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The $n$ by $r$ right-hand side matrix $B$.

### 5.2 Optional Input Parameters

1: **n** – **int32 scalar**

*Default*: The dimension of the array **d**.

$n$, the order of the matrix $A$.

*Constraint*: $\mathbf{n} \geq 0$.

2: **nrhs_p – int32 scalar**

*Default*: The second dimension of the array **b**.

$r$, the number of right-hand sides, i.e., the number of columns of the matrix $B$.

*Constraint*: **nrhs_p** $\geq 0$.

### 5.3 Input Parameters Omitted from the MATLAB Interface

ldb

### 5.4 Output Parameters

1: **d**$(*)$ **– double array**

**Note**: the dimension of the array **d** must be at least $\max(1, \mathbf{n})$.

The $n$ diagonal elements of the diagonal matrix $D$ from the factorization $A = LDL^{\mathrm{T}}$.

2: **e**$(*)$ **– double array**

**Note**: the dimension of the array **e** must be at least $\max(1, \mathbf{n} - 1)$.

The $(n - 1)$ subdiagonal elements of the unit bidiagonal factor $L$ from the $LDL^{\mathrm{T}}$ factorization of $A$. (**e** can also be regarded as the superdiagonal of the unit bidiagonal factor $U$ from the $U^{\mathrm{T}}DU$ factorization of $A$.)

3: **b**(**ldb**,$*$) **– double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

If **info** $= 0$, the $n$ by $r$ solution matrix $X$.

4: **info – int32 scalar**

**info** $= 0$ unless the function detects an error (see Section 6).

## 6 Error Indicators and Warnings

Errors or warnings detected by the function:

**info** $= -i$

If **info** $= -i$, parameter $i$ had an illegal value on entry. The parameters are numbered as follows:

1: **n**, 2: **nrhs_p**, 3: **d**, 4: **e**, 5: **b**, 6: **ldb**, 7: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

**info** $> 0$

If **info** $= i$, the leading minor of order $i$ is not positive-definite, and the solution has not been computed. The factorization has not been completed unless $i = \mathbf{n}$.

## 7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and $\epsilon$ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \left\|A^{-1}\right\|_1 \|A\|_1$, the condition number of $A$ with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* 1999 for further details.

f07jb is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, f04bg solves $Ax = b$ and returns a forward error bound and condition estimate. f04bg calls f07ja to solve the equations.

## 8 Further Comments

The number of floating-point operations required for the factorization of $A$ is proportional to $n$, and the number of floating-point operations required for the solution of the equations is proportional to $nr$, where $r$ is the number of right-hand sides.

The complex analogue of this function is f07jn.

## 9 Example

```
d = [4;
     10;
     29;
     25;
     5];
e = [-2;
     -6;
     15;
     8];
b = [6;
     9;
     2;
     14;
     7];
[dOut, eOut, bOut, info] = f07ja(d, e, b)
```

```
dOut =
     4
     9
    25
    16
     1
eOut =
   -0.5000
   -0.6667
    0.6000
    0.5000
bOut =
    2.5000
    2.0000
    1.0000
   -1.0000
    3.0000
info =
           0
```